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DETERMINATION OF THE OUTPUT CHARACTERISTICS OF ELECTRON TUBE GENERATORS

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I. INTRODUCTION

Within the last three years the experimental development of the three-electrode electron tube as a generator of alternating currents has been carried out to such an extent that the device is at present the standard source of supply for radio telephone and telegraph circuits in all cases where extremely high power is not required. At the same time no adequate methods have been evolved whereby the efficiency and power output of such a generator can be stated with any degree of accuracy in terms of the electrical constants of the tube itself. Several analyses of the general operation of electron tube generators are now available.[1] But none of these are accurate enough and at the same time comprehensive enough to allow a quantitative prediction from the two important factors in power-tube operation—plate voltage and filament emission—of the alternating-current power which can be developed in a radio-frequency circuit of known resistance, inductance, and capacity. Power tubes are rated in watts output and watts input from empirical data upon circuits adjusted to particular settings, experimentally determined, for maximum power or maximum efficiency. Such ratings give no intimation of the power which will be developed by the tube if any constant of the output circuit be changed.

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[1] J. Bethenod, Lum. Elect., Dec. 9, 1916.
G. Vallauri, L'Elettrotecnica: IV, No. 3, pp. 18, 19; 1917.
L. A. Hazeltine, Proc. I. R. E., April, 1918.
T. Kikuchi, Proc. Physico-Mathematical Society of Japan, February, 1919.

In this paper a method of analysis will be presented by means of which it is possible to design a circuit to obtain maximum output from a given tube, or, conversely, to select a tube which will furnish with reasonable efficiency its maximum power to a particular output circuit.

II. OSCILLATING TUBE

The explanation of the phenomena taking place in connection with the oscillations generated by an electron tube can well be illustrated by the diagrammatic sketch in Fig. 1. This represents the so-called Hartley circuit which is typical of the circuits used for the purpose of generating oscillations. The grid and filament of the tube in this particular circuit are connected to the branched circuit containing inductance and capacity, constituting the oscillatory circuit, in such a manner as to include a portion of the inductance of that circuit, that is the coil L_g. Likewise between the plate and filament of the tube is included the coil L_p.

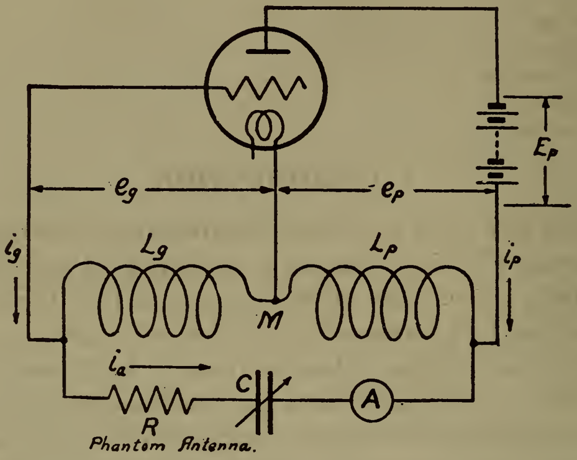

FIG. 1.—*Direct-coupled generating circuit*

Suppose, first, that the tube is not generating oscillations. Under the action of the plate battery a steady current will flow from the plate to the filament inside of the tube and from filament to plate through the coil L_p in the external circuit. The magnitude of this current will vary with the voltage of the grid; it can become zero when the grid is highly negative but can not reverse in direction. Also there can be a flow from grid to filament inside the tube, the current returning from filament to grid through the coil L_g in the external circuit. This latter current is appreciable only when the grid is positive with respect to the filament. Except for effects which are extraneous to this treatment, this current is also unidirectional.

When the tube is in the oscillating condition these currents will not be steady, but will become pulsating. The pulsating currents generated by the tube enter the circuit from the filament and leave it through the plate and grid connections; they are pulsating, and not alternating, on account of the unidirectional conductivity between filament and plate and filament and grid. The circuit L_g L_p C is resonant to the fundamental constituents of these pulsating currents, and an oscillatory current is generated which circulates around this branched circuit, flowing in series through the condenser C and the coils L_p and L_g. This current, which will be called the output current, can be many times greater in amplitude than either of the pulsating currents. The pulsations in the steady current, which flows during the static condition from the filament to the plate, are caused by periodic variations in the potential of the grid with respect to the filament; these variations in grid potential are induced in the grid coil L_g by the output current. There is a similar voltage induced by the output current across the plate coil L_p. It is true of this circuit, and typical of any circuit for generating oscillations, that during that portion of the cycle of the output current when the grid is positive with respect to the filament as a result of the voltage induced in the coil L_g, the voltage drop between the plate and filament connections (across the coil L_p) is such as to oppose the voltage of the plate battery and hence to reduce the voltage acting between filament and plate in the tube. During the other part of the cycle, when the grid is negative with respect to the filament, the voltage acting between plate and filament is increased above that of the steady voltage of the plate battery. During the portion of the cycle when the grid is positive with respect to the filament current flows within the tube between the grid and filament, and this current increases as the grid becomes more positive. The direction of the current flow is in the direction of the emf, that is, from grid to filament inside the tube and from filament to grid outside of the tube. Further, as the grid becomes positive with respect to the filament, there is a resultant increase in the current flow between the plate and filament of the tube, even though the plate voltage on the tube is being reduced. This increase is limited, when the stable oscillating condition has been reached, by the saturation effect, which may occur at lower values of plate current than that corresponding to the total filament emission, owing to the loss of electrons to the grid.

As has been stated, the plate current wave is distorted at the other extreme of the cycle—that is, when the grid is negative—by rectification effects; moreover, the grid current is always pulsating, and is zero for a considerable part of a cycle, while the grid is negative. Consequently the waves of current supplied to the circuit between *F* and *P* and *F* and *G* (Fig. 2) are each composed of a direct or average constituent, a fundamental constituent corresponding in frequency to that of the output current, and a number of higher frequency or harmonic constituents.

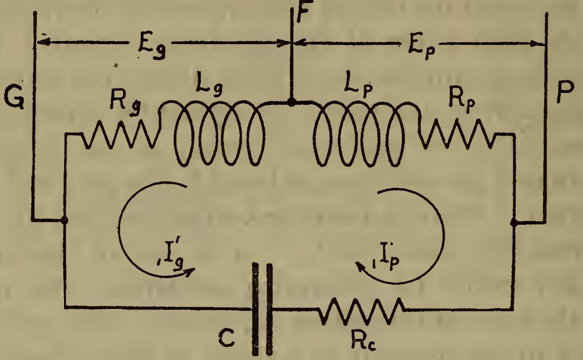

FIG. 2.—*Output circuit*

The useful oscillating output current depends neither upon the direct or average values of the plate and grid currents nor upon the multiple frequency constituents; it is determined solely by the fundamental constituents of these currents, to which the same considerations as regards direction and phase relations apply as have been roughly stated with regard to the distorted current waves.

Thus, speaking only in terms of the useful current constituents, a sinusoidal, alternating current flows in the grid circuit in phase with the alternating voltage across the coil L_g, and therefore represents a withdrawal of power from the output circuit, which power is expended within the tube. On the other hand, a sinusoidal, alternating constituent of plate current flows in opposition to the voltage across the coil L_p; this means that power is being supplied to the output circuit from the plate circuit of the tube. As will appear later, the impedance of the output circuit to all frequencies that are harmonic multiples of the fundamental is very great;

hence there are no appreciable multiple frequency constituents of current circulating in the output circuit, and the alternating voltages across the coils L_p and L_g are in all cases practically sinusoidal. Consequently the useful power supplied by the tube can be determined in terms of the alternating voltage across L_p and the fundamental constituent of plate current. If we could neglect the grid current, this would be the power available for dissipation in the resistances. As the output current increases the amplitudes of the alternating voltages across the plate and grid coils increase proportionately. The alternating grid current increases more and more rapidly, as the amplitude of the plate voltage becomes larger. On this account the power loss to the grid increases. The power supplied by the plate increases with increasing plate voltage, but as the grid voltage increases, the effective saturation current is reached when the grid is positive, and the plate current becomes zero for an appreciable part of the cycle when the grid is negative; consequently a continued increase in the amplitude of the output current results chiefly in an increase in the harmonic constituents of plate current without greatly increasing the fundamental. Obviously, then, a condition of stability ensues when the power supplied by the fundamental of plate current minus the power dissipated by the fundamental of grid current is just equal to the power dissipated by the output current in R_g, R_p, and R_c.

III. DERIVED CHARACTERISTIC

The first quantitative measurements that must be made upon a tube are measurements of the plate and grid currents which flow when different plate and grid voltages are applied to the tube. It is evident from the arrangement of the output circuit, Fig. 2, that ${}_1e_p$ and ${}_1e_g$, the instantaneous values of the voltage induced by the output current in the plate and grid coils, are not only opposite in sign with respect to the neutral filament, but are at all times in the same ratio of magnitudes as the ratio of inductance of the plate and grid coils. If we define $n = L_p/L_g$, then at any instant during a cycle of output current ${}_1e_p = n\ {}_1e_g$, assuming these voltages to be caused entirely by the output current. Thus it is possible to apply steady voltages to a tube, to vary these voltages step by step maintaining the same ratio as during an actual oscillation and to measure the instantaneous values which the plate and grid currents will assume during an oscillation.

Curves showing the plate and grid currents as explicit functions of the grid voltage, taking account of the simultaneous changes in plate voltage which would occur if these currents supplied an output circuit having a given ratio of coupling reactances are called the derived characteristic of a tube. This term was proposed by Hazeltine,[2] who also originated the idea of a constant ratio of plate and grid voltages while a tube is in oscillation. According to his method the characteristic is plotted by making successive changes in a steady value of e_g, the voltage applied to the grid of the tube, at the same time making changes in e_p, the voltage applied to the plate, opposite in direction and equal in magnitude to $n\ e_g$, and by measuring the resulting values of i_g and i_p. Such a derived characteristic is shown in Fig. 3, plotted for a type *VT*–16 tungsten-filament tube starting from the operating point determined by a steady plate voltage of 300 and grid voltage of zero. A battery of small resistance was connected between filament and grid, and with each increase in the potential of

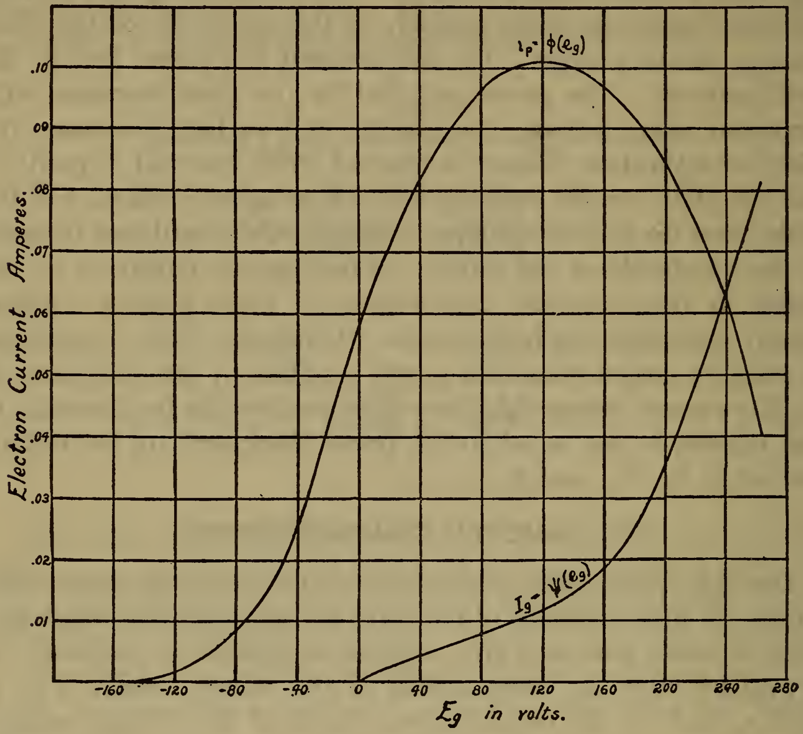

FIG. 3.—*Derived characteristic*

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[2] L. A. Hazeltine, loc. cit.

the grid above that of the filament the voltage of the plate battery was decreased by an equal amount, while for each reduction in the grid voltage below that of the filament the plate-battery voltage was increased. Hence this characteristic is applicable to any circuit in which $n=1$.

The value of a derived characteristic obtained in this manner as an indication of the grid and plate currents during an actual oscillation depends upon (1) the equality of the output current through the two branches of the output circuit from filament to plate; and (2) the constancy with which a phase difference of 180° is maintained between voltages of plate and grid with respect to the filament.

In the subsequent sections the following points will be discussed:

1. A method for obtaining from the Hazeltine derived characteristic the power characteristics of the tube.
2. The application of these power characteristics in determining the current output in a circuit of known capacity and resistance.
3. Experimental results upon an oscillating circuit justifying both the Hazeltine characteristic and the application thereof.

IV. POWER OUTPUT

In the circuit shown in Fig. 1 i_p, i_g, and i_a are the instantaneous values of the plate, grid, and antenna or output currents, respectively, while E_b is the steady voltage of the plate battery and ${}_1e_g$ and ${}_1e_p$ are the instantaneous values of the alternating voltages across the grid and plate coupling coils. R_p and R_g, the resistances of the coupling coils, are small compared with the reactances of these coils, so the alternating plate and grid voltages can be considered as due only to the reactances of these coils, justifying the definition of n by the ratio L_p/L_g.

Let a derived characteristic of the tube be plotted for this value of n. Then i_p and i_g are known as explicit functions of e_g.

$$\begin{aligned} i_p &= \phi(e_g) \\ i_g &= \psi(e_g) \end{aligned} \qquad (1)$$

Make the assumption, which will be justified later, that i_a is undistorted and hence ${}_1e_g$ is a pure sinusoid, of the form:

$${}_1e_g = {}_1E_g \sin\omega t$$

Then $$i_p = \phi({}_1E_g \sin \omega t)$$

and $$i_g = \psi({}_1E_g \sin \omega t)$$

Whatever the form of the ϕ and ψ functions, the wave forms of i_p and i_g will be symmetrical with respect to $\pi/2$ from $\omega t = 0$ to $\omega t = \pi$, and also with respect to $3\pi/2$ from $\omega t = \pi$ to $\omega t = 2\pi$. This indicates that the phase angles of all harmonic constituents of i_p and i_g are alternately 0 and 90° with respect to the fundamental, the frequency of which is that of the grid voltage, ω. That is:

$$\begin{aligned} i_p &= {}_0I_p + {}_1I_p \sin \omega t + {}_2I_p \cos 2\omega t + \cdots {}_{2k-1}I_p \sin (2k-1)\omega t + \\ &\qquad {}_{2k}I_p \cos 2k\omega t - - \\ i_g &= {}_0I_g + {}_1I_g \sin \omega t + {}_2I_g \cos 2\omega t + \cdots {}_{2k-1}I_g \sin (2k-1)\omega t + \\ &\qquad {}_{2k}I_g \cos 2k\omega t - - \end{aligned} \qquad (3)$$

The current in the output circuit will be of the form:

$$\begin{aligned} i_a &= {}_1I_a \sin \omega t + {}_2I_a \cos 2\omega t \cdots {}_{2k-1}I_a \sin (2k-1)\omega t + \\ &\qquad {}_{2k}I_a \cos 2k\omega t - - \end{aligned} \qquad (4)$$

In order for ${}_1e_g$ to be approximately sinusoidal the amplitude of any harmonic constituent, ${}_kI_a$, must be small compared with the amplitude of the fundamental ${}_1I_a$. It is found in practice that the instantaneous values of i_p, given by $i_p = \phi(L_g \omega_a I_1 \sin \omega t)$ differ from those given by $i_p = \phi[L_g\omega({}_1I_a \sin \omega t + {}_2I_a \cos 2\omega t - -)]$ by an amount less than the experimental errors introduced in plotting the derived characteristic which determines the ϕ function. (Equation 1.)

The amplitude of any constituent of the output current, ${}_kI_a$, can be calculated in terms of the amplitude of the same constituent of the plate and grid currents, ${}_kI_p$ and ${}_kI_g$, by consideration of the impedance of the output circuit to that frequency, $k\omega$. For the present, however, we shall consider only the constituents which determine the input and the useful output. These constituents are ${}_0I_p$, ${}_1I_p$, and ${}_1I_g$, the direct component of the plate current, the sinusoidal fundamental constituent of the plate current, and the fundamental constituent of the grid current, respectively. Assuming sinusoidal plate and grid voltages, these constituents can be calculated directly from the derived characteristic by any method of approximate harmonic analysis. Fig. 4 shows ${}_1I_p$, ${}_0I_p$, and ${}_0I_g$ as functions of ${}_1E_g$, the amplitude of the grid voltage. The points on these curves were obtained for a given value of ${}_1E_g$ by calculation from a number of measured ordinates of the characteristic, Fig. 3, equally spaced throughout a cycle which i_g and i_p would pass as ${}_1E_g \sin \omega t$ traverses a complete cycle. It is not necessary to plot the actual current waves in order to space these ordinates equally. If 18 ordinates be used, they can be measured directly upon the characteristics at values of e_g given by ${}_1E_g \sin \pi/9$, ${}_1E_g \sin 2\pi/9$, ${}_1E_g \sin \pi/3$, etc.

The curves of Fig. 4 are sufficient to indicate the power output and efficiency which can be obtained from this tube in any output circuit for which $n=1$, the tube being operated at 300 volts steady plate potential and 1.35-ampere filament current.

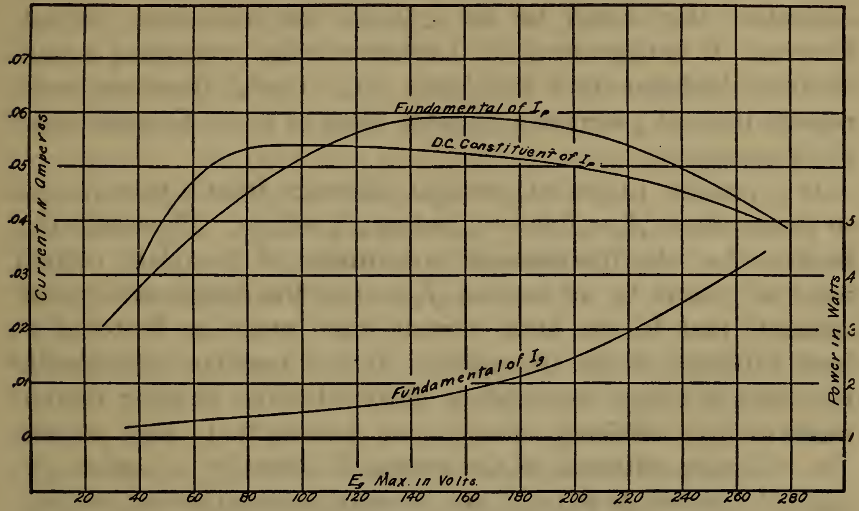

FIG. 4.—*Fundamental current amplitudes*

The effective power supplied by the plate battery is $E_b \times {}_0I_p$. ${}_0I_p$ is actually the reading of a d. c. ammeter connected in series with the plate battery, since all of the alternating constituents are pure sinusoids, symmetrical with respect to ${}_0I_p$. The power output from the tube can be determined either in terms of the output current and the resistance $R=R_p+R_g+R_c$, or in terms of the alternating plate and grid voltages associated with the fundamental constituent of the plate and grid currents.

In terms of the plate and grid voltage the effective power output is

$$P_o=\frac{1}{2}\left({}_1E_p\,{}_1I_p-{}_1E_g\,{}_1I_g\right)$$

Substituting ${}_1E_p=n_1E_g$:

$$P_o=\frac{{}_1E_p}{2}\left({}_1I_p-\frac{1}{n}{}_1I_g\right)=\frac{n_1E_g}{9}\left({}_1I_p-\frac{1}{n}{}_1I_g\right) \tag{5}$$

The power drawn from the plate battery is

$$P_i=E_b \times {}_0I_p \tag{6}$$

Hence the efficiency, for any inductively coupled output circuit, is

$$E_{\mathrm{ff}} = 0.5 \frac{{}_1E_{\mathrm{p}}}{E_{\mathrm{b}}} \Bigg(\frac{{}_1I_{\mathrm{p}} - {}_1I_{\mathrm{g}}/n}{{}_0I_{\mathrm{p}}} \Bigg(= 0.5 \frac{{}_1E_{\mathrm{g}}}{E_{\mathrm{b}}} \left(\frac{n_1I_{\mathrm{p}} - {}_1I_{\mathrm{g}}}{{}_0I_{\mathrm{p}}} \right) \qquad (7)$$

It might be supposed that the maximum possible power is obtained from the tube when the plate voltage is reduced to zero at each oscillation; that is, when ${}_1E_{\mathrm{p}} = E_{\mathrm{b}}$. If ${}_1I_{\mathrm{g}}$ could be neglected, this would be the criterion for maximum output. However, it is characteristic of electron tubes employing a pure electron discharge that the factor $({}_1I_{\mathrm{p}} - 1/n_1I_{\mathrm{g}})$ decreases more rapidly than nE_{g} increases, for any value of n, as ${}_1E_{\mathrm{p}}$ approaches E_b in amplitude.

It is possible to get 50 per cent efficiency from a tube if n be so chosen that $({}_1I_{\mathrm{p}} - {}_1I_{\mathrm{g}}/n) = {}_0I_{\mathrm{p}}$ when ${}_1E_{\mathrm{g}} = E_{\mathrm{b}}/n$. Physically this implies that the fundamental constituent of the plate current must be greater by an amount ${}_1I_{\mathrm{g}}/n$ than the direct-current constituent; that is, the plate current wave must be flattened at both extremes of the alternation. It is a familiar experimental fact that a highly distorted or flattened wave of plate current leads to high efficiency, though not necessarily to high output. The ultimate criterion of the output is given by equation (5), and it is impossible to make any accurate generalizations concerning maximum output without analytical expressions for the characteristic surfaces of both plate and grid currents. It is conceivable that with values of n greater than 1, with a high plate-battery voltage, and with a negative potential applied to the grid, an efficiency greater than 50 per cent might be obtained.

By reference again to Fig. 4 it can be seen that for this particular tube, working at a plate voltage of 300 and supplying any output circuit for which $n = 1$, maximum output will be obtained at plate and grid voltages considerably smaller than E_{o}. The maximum value of the function $\frac{n_1E_{\mathrm{g}}}{2}({}_1I_{\mathrm{p}} - {}_1I_{\mathrm{g}}/n)$ occurs at ${}_1E_{\mathrm{g}} = 190$, when $P_{\mathrm{o}} = 4.37$ watts. The maximum efficiency occurs at approximately the same point and is 29.5 per cent. It does not follow from this that the maximum efficiency occurs at the point of maximum output for any other value of n or of E_{b}. Without changing E_{b} the tube could be operated more efficiently with a higher value of n. In general, it is found that a tube operated at high plate voltages yields a large fundamental of plate current relative to the direct constituent, owing to the increased flattening of the wave form due to saturation and rectification effects.

Hence operation at higher voltage on the plate of this tube would give increased output, owing to the decrease in ${}_1I_g$ which accompanies high values of E_b, and increased efficiency, due both to this and to the saturation effect.

V. CURRENT OUTPUT

The current characteristics of the generator, such as are shown on Fig. 4, are entirely adequate for the prediction of the maximum power and efficiency that a tube is capable of delivering. In order to determine what constants of the output circuit will give the maximum output, an analysis of this output circuit is necessary. The plate and grid voltages depend upon the fundamental constituent of the output current ${}_1I_a$, which, in turn, depends upon ${}_1I_p$, the fundamental constituent of the plate circuit. In order to calculate the actual current in the output circuit which will dissipate the power furnished by the tube, this circuit must be correlated with the tube circuit so as to express the output characteristic, using as a parameter a variable of the output circuit instead of one of the exciting voltages. Thus we can derive from the general output characteristic of the tube a particular supply characteristic for the output circuit.

Consider the circuit shown in Fig. 2, which is merely the output circuit of the inductively coupled system shown in Fig. 1. Speaking again only in terms of the sinusoidal fundamental constituents ${}_1I_p$ enters the circuit at P, divides between the path L_p R_p on one side and the path R_c C R_g L_g on the other side, leaving the circuit at F. Similarly, ${}_1I_g$ enters the circuit at G, divides between R_g, L_g, and C, R_c, R_p, L_p, leaving the circuit at F. The fundamental of plate current, ${}_1I_p$, maintains a circulating current in the output circuit, ${}_1I_p'$, in comparison with which ${}_1I_p$ itself may be negligibly small, if the series resistance, $R=R_g+R_p+R_c$, be small compared with the effective resistance of the divided circuit between F and P. If the current distribution in the circuit be determined only by the reactances—that is, if the resistance of each branch of the circuit be small compared with the reactance of that branch—the current ${}_1I_p'$ will be equal in all parts of the circuit. Similarly, the fundamental of grid current maintains in the parallel circuit between G and F a circulating current ${}_1I_g'$, which opposes ${}_1I_p'$ in all branches of the circuit. The useful output current can be thought of as the difference between these

$${}_1I_a={}_1I_p'-{}_1I_g'$$

A convenient method of calculating the output current is the loss method, which was employed by Hazeltine[3] in considering briefly the entire distorted plate current wave. The net power supplied to the circuit, taking account of the grid losses, expressed above by the opposition of ${}_1I_g'$ to ${}_1I_p'$, is given by equation (5):

$$P_o = \frac{n_1 E_g}{2}\left({}_1I_p - I_g/n\right)$$

P_o can be obtained graphically as a function of E_g from the current characteristic (such as Fig. 4).

$$P_o = f\,({}_1E_g) \tag{8}$$

But

$$\begin{aligned} {}_1E_g &= L_g \omega_1 {}_1I_a \\ &= \frac{L_g}{\sqrt{(L_g + L_p + 2M)C}}\,{}_1I_a \end{aligned} \tag{9}$$

Moreover,

$$P_o = {}_1I_a^2 R. \tag{10}$$

where $R = R_p + R_g + R_c$. Eliminating ${}_1I_a$ by substitution from (10) in (9)·

$${}_1E_g = L_g \sqrt{\frac{P_o}{R\,C\,(L_p + L_g + 2M)}} \tag{11}$$

also

$$P_o = f\,({}_1E_g) \tag{8}$$

A simultaneous graphical solution of these two equations gives P_o, the power output, as a function either of the resistance or of the capacity in the output circuit.

$$P_o = f'\,(C),\ R = \text{const.}$$
$$P_o = f''\,(R),\ C = \text{const.}$$

Since ${}_1I_a = \sqrt{\frac{P_o}{R}}$ the output current follows immediately as a function of the variables of the output circuit.

In making a numerical proof of the foregoing theory for the tube described by the characteristics of Fig. 4 a slightly different method of attack was used in order to facilitate the calculation of the various harmonic constituents of the output current. The principal harmonics have been calculated and measured in order to indicate the insignificance of the error involved in assuming sinusoidal plate and grid voltages for a determination of the output characteristics.

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[3] Loc. cit.

In the circuit of Fig. 2 let $L_p = L_g$; then $n = 1$ and Fig. 4 leads to an expression of the power which the tube will develop in this circuit. As a further simplification make $M = O$, the mutual between L_p and L_g. Then the equation for the power output reduces to

$$P_o = \frac{{}_1E_g}{2}({}_1I_p - {}_1I_g),$$

and we can consider this circuit as being driven by an effective supply current, ${}_1I_e$, which is the difference between the fundamental of plate current and the fundamental of grid current.

$${}_1I_e = {}_1I_p - {}_1I_g.$$

Similarly, for any harmonic constituents of the plate and grid currents, the effective current is

$${}_kI_e = {}_kI_p - {}_kI_g.$$

Any constituent of the circulating output current can be found as the high-amplitude components of ${}_kI_e$ which flow through either branch of the divided circuit between F and P (Fig. 2). Obviously if $\sqrt{R_p^2 + L_p^2\omega^2} = \sqrt{R_g^2 + L^2{}_g\omega^2}$ the effective supply current can be treated as if applied to the output circuit between F and G or between F and P. In either case, if we assume that the resistances of the separate branches are small compared with the reactances, a consideration of the relative impedances of the inductance branch and the inductance-capacity branch of the output circuit leads to the equality of any individual constituent of the output current in all parts of the output circuit, a fact mentioned previously in connection with the fundamental alone. If ${}_kI_a$ be the kth frequency constituent of the output current flowing through the capacity branch of the circuit, then ${}_kI_a$ is related to the kth constituent of the effective supply current by the relation

$$\frac{{}_kI_a}{{}_kI_e} = \frac{kL\omega}{\sqrt{R^2 + \left(2kL\omega - \frac{1}{kC\omega}\right)^2}} \qquad (12)$$

assuming that the total impedance of either of the coupling coils is $kL\omega$. This is justifiable even when $k = 1$. Substituting $\omega = \frac{1}{\sqrt{2LC}}$ this reduces to

$$\frac{{}_kI_a}{{}_kI_e} = \frac{k}{\sqrt[2]{\left(k - \frac{1}{k}\right)^2 + \left(\frac{R}{2Lk\omega}\right)^2}}$$

For all harmonic constituents $\left(\frac{R}{2Lk\omega}\right)^2$ is insignificant compared with $\left(k-\frac{1}{k}\right)^2$ and this reduces to

$$ {}_kI_a = \frac{k^2}{2(k^2-1)}\,{}_kI_e \tag{13}$$

For the fundamental constituent $k=1$, $\frac{R}{2L\omega}$ becomes important, and the output current is given by

$$\begin{aligned} {}_1I_a &= \frac{2L\omega}{R}\,{}_1I_e \\ &= \frac{1}{R}\sqrt{\frac{2L}{C}}\,{}_1I_e \end{aligned} \tag{14}$$

An immediate deduction from these expressions is that the ratio of the fundamental in the antenna to any harmonic is practically inversely proportional to the antenna resistance. In other words, the impedance of the output circuit to the fundamental constituent of the supply current is merely the resistance of the circuit, while the resistance forms an insignificant part of its impedance to all harmonics. A small change in R thus makes a large change in the fundamental constituent relative to the corresponding change in the harmonic.

${}_1I_e$ is known empirically from the curves of Fig. 4 as a function of the grid voltage amplitude: ${}_1I_e = f({}_1E_g)$. But ${}_1E_g = L\omega\,{}_1I_a = \sqrt{\frac{L}{2C}}\,{}_1I_a$. Eliminating ${}_1I_a$ between this and equation (14):

$$\left\{\begin{aligned} {}_1I_e &= \frac{2RC}{L}\,{}_1E_g && (15)\text{a} \\ {}_1I_e &= f({}_1E_g) && (15)\text{b} \end{aligned}\right.$$

also—

Voltage characteristics of the output circuit showing E_g and E_p as functions of the resistance and capacity are furnished by graphical solutions of these equations. Fig. 5*a* shows such solutions for constant resistance and variable capacity and Fig. 5*b*, for constant capacity and variable resistance, for a circuit in which $L = 0.365$ millihenries, $R = 2.20$ ohms, and $C = 5.35$ millimicrofarads. These inductance and capacity values give a range of fundamental wave lengths of 3000 to 10 000 meters, which are large enough to make the capacitive reactance of the plate and grid coils insignificant compared with their inductive reactance. The graphical solutions are accomplished by plotting equation (15b) from the output characteristic of Fig. 4, by direct subtraction of ${}_1I_g$ from ${}_1I_p$ and by plotting equation (15a) for a number of values of C, at a constant value of R, and for a number of values of R at a constant value of C. The points of intersection of these straight lines with the empirical function (15b) give E_g as a function of the output capacity, and again, as a function of the output resistance. These are shown upon Fig. 6, the curves 6*a* being derived from 5*a*, and 6*b* from 5*b*.

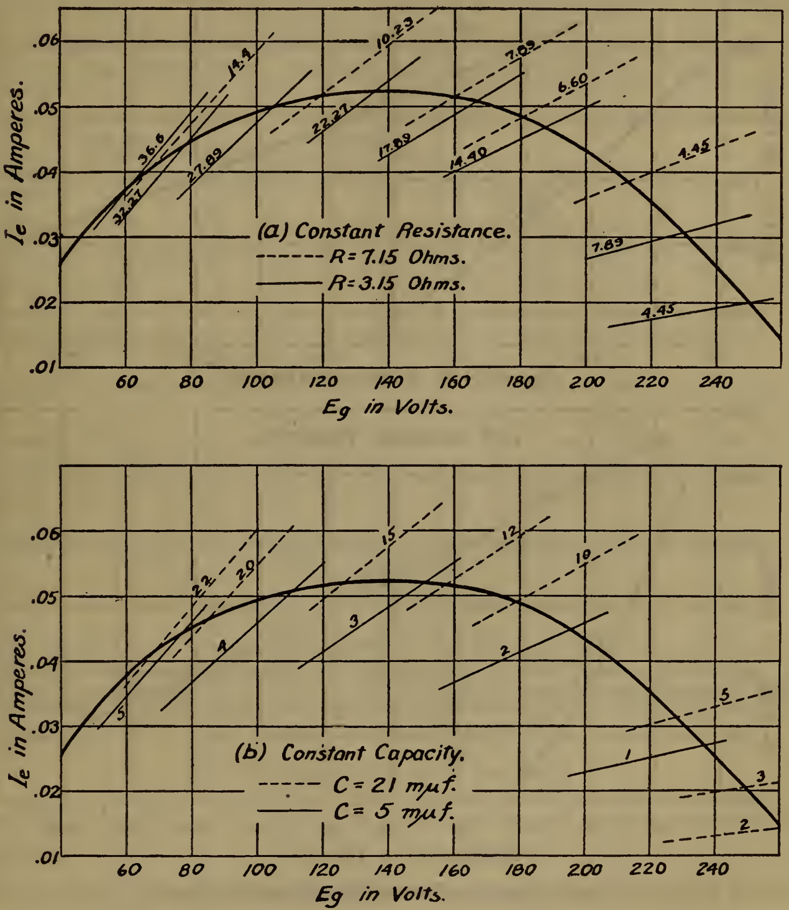

Fig. 5.—*Graphical solutions*

With the voltage characteristics as the connecting link between tube and output circuit, the effective supply current ${}_1I_e$ can be shown as a function either of the resistance or of the capacity in the output circuit; this might be termed a current supply characteristic for this particular output circuit, contrasted with the output characteristics of the generator (Fig. 4) which are applicable to any output circuit. The current supply characteristic is shown in Fig. 7 for two values of antenna resistance, $R=7.15$ ohms, and $R=3.15$ ohms, and variable capacity. The effect of increasing the antenna resistance is to make the maximum output

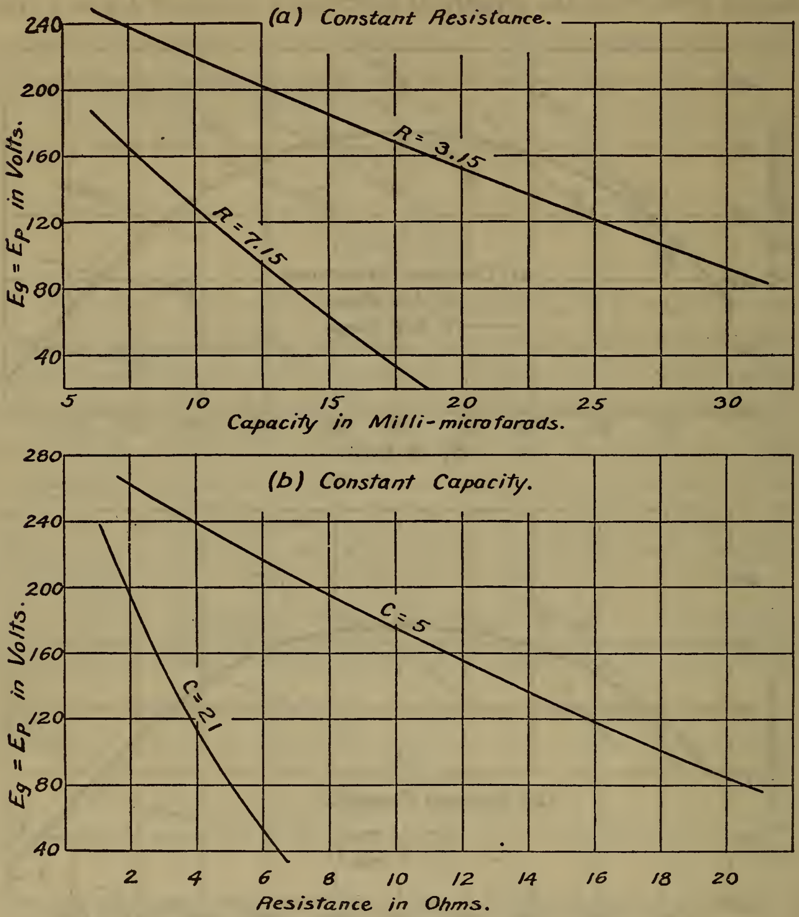

FIG. 6.—*Voltage characteristics of output circuit*

from the tube occur at lower capacity values. Upon Fig. 7 are shown, for purposes of comparison, the numerical values of ${}_2I_e$, ${}_3I_e$, ${}_4I_e$, ${}_5I_e$. The harmonic constituents of plate and grid currents were calculated from the derived characteristic of Fig. 3 following the same method employed for the fundamental constituents. ${}_kI_p$ and ${}_kI_g$ were subtracted algebraically and the resulting ${}_kI_e$ plotted as a function of C for one value of R, $R=3.15$ ohms, by virtue of the voltage characteristic, Fig. 6*a*.

The output currents for various capacity values follow immediately from the supply characteristic by substitution in equations (13) and (14). In Fig. 8 are shown theoretical curves, of output current, calculated in this manner. The points marked by circles show actual hot-wire ammeter readings, multiplied by $\sqrt{2}$, observed in a circuit having these constants, and supplied by the tube for which output characteristics shown on Fig. 4 had been obtained. The calculated and experimental results check quite closely, in view of the difficulties encountered in reproducing exactly the operating condition of the tube from one set of measurements to the next.

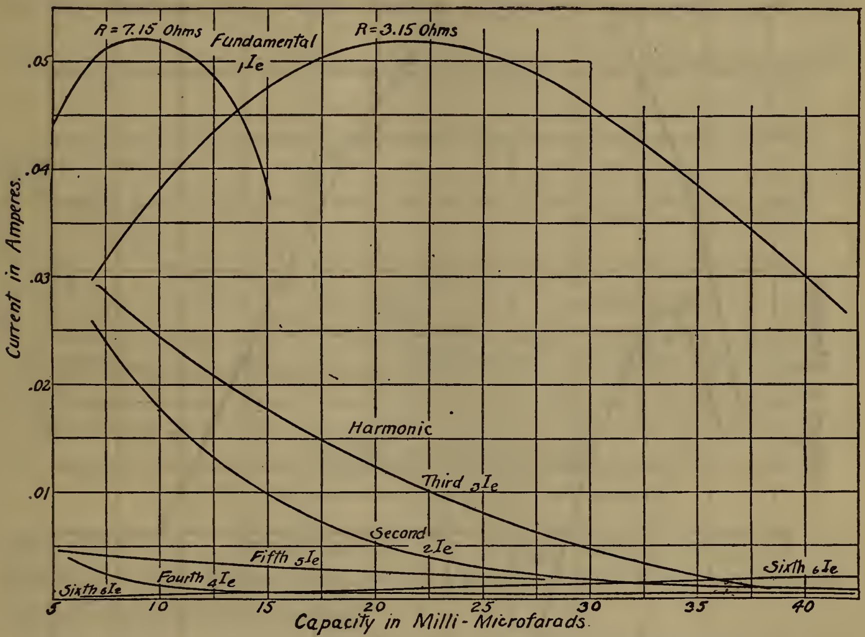

FIG. 7.—*Supply current characteristics*

It is a matter of experience that in any output circuit in which the ratio $\frac{R}{L_a\omega} < 1$ the harmonics are practically insignificant in their effect upon a hot-wire ammeter. Consequently the readings of such an instrument can be taken as a true indication of the amplitude of the fundamental.

Upon Fig. 8 are shown calculated amplitude curves for the double and triple harmonics at $R = 3.15$ ohms. The highest value reached by any harmonic is 3 per cent of the fundamental at points well below maximum output. At maximum output all harmonics are entirely negligible.

Upon Fig. 9 are shown calculated curves for the double, triple, quadruple, and quintuple frequency harmonics, at a resistance of 3.15 ohms. These curves were calculated by substitution from Fig. 7 in equation (13). Measurements of the harmonics were made by the following method: A standard wave length circuit with a 4-ohm vacuum thermoelement was coupled to a small wire loop connected in series with the output capacity. The wave meter was tuned to resonance with the various harmonic frequencies and the currents induced at these frequencies measured; then with the same coupling and a suitable noninductive resistance in series with the thermoelement the wave meter was tuned

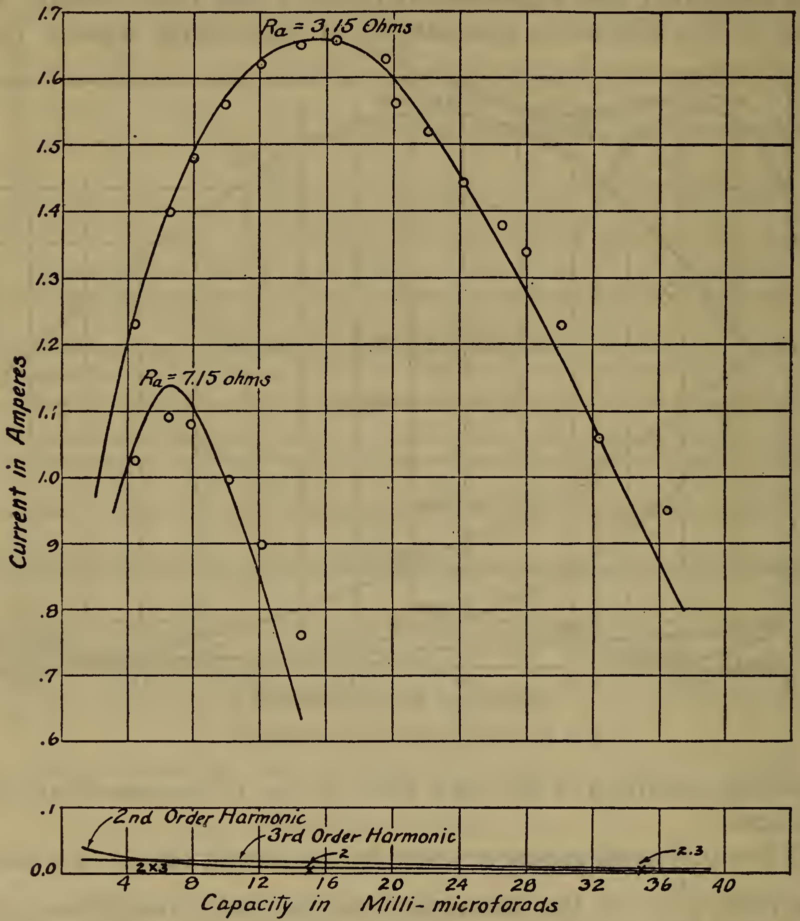

FIG. 8.—*Output currents*

to resonance with the fundamental. The resistance of the standard circuit being accurately known, the voltages induced in this circuit at the fundamental and at the harmonic frequencies could be computed. These voltages indicate the relative amplitudes of the harmonic constituents of the output current, provided that account is taken of the change in frequency from fundamental to harmonic. The relative amplitudes of the harmonics with respect to the fundamental being known, their absolute amplitudes could be calculated from the deflections of the hot-wire ammeter in the output circuit, assuming these to be due entirely to the fundamental. The double and triple harmonics are the only ones of any importance in the output circuit, owing to the extremely high impedance of this circuit to the higher multiples of the fundamental frequency.

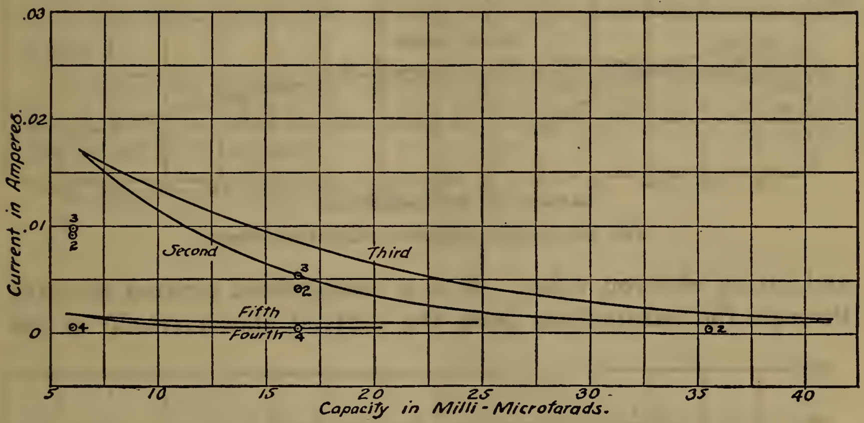

FIG. 9.—*Harmonic constituents of output currents*

Upon Figs. 10 and 11 are shown power and efficiency curves for the output circuit, giving the input on the tube and the output power as functions of the capacity for constant resistance, and of the resistance, for constant capacity. The calculated curves were obtained by multiplying ${}_1I_e$ and ${}_1E_p$ as obtained from Fig. 4, and expressing these products as functions of the constants of the output circuit by the use of the voltage characteristics (Fig. 6). This process is equivalent to taking the theoretical expression for ${}_1I_a$, equation (13), squaring, and multiplying by the resistance. The measured values of power were calculated from the hot-wire ammeter readings shown on Fig. 8.

It must not be inferred from the foregoing discussion that this analysis is recommended for use in toto as a method for testing and rating electron tubes. It is a cumbersome process to carry through the calculations from the derived characteristic of the tube to the output current as a function of the output resistance and capacity.

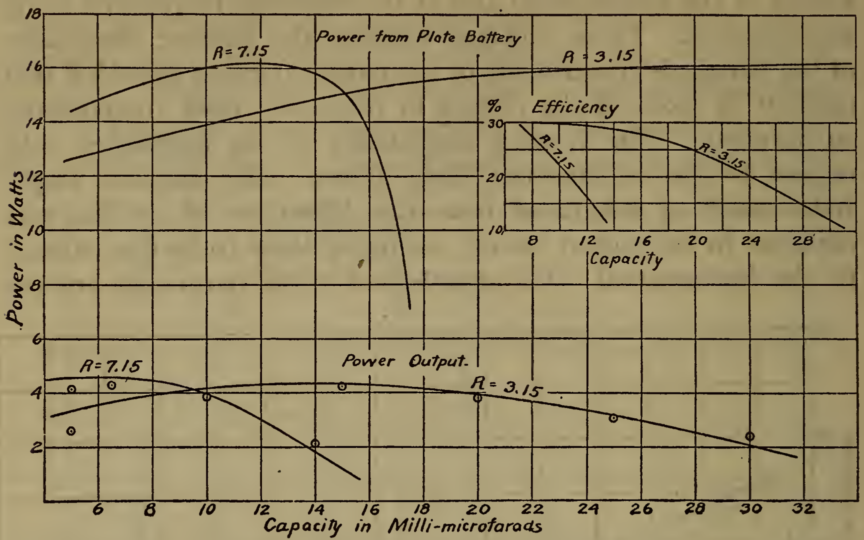

FIG. 10.—*Power output—Constant resistance*

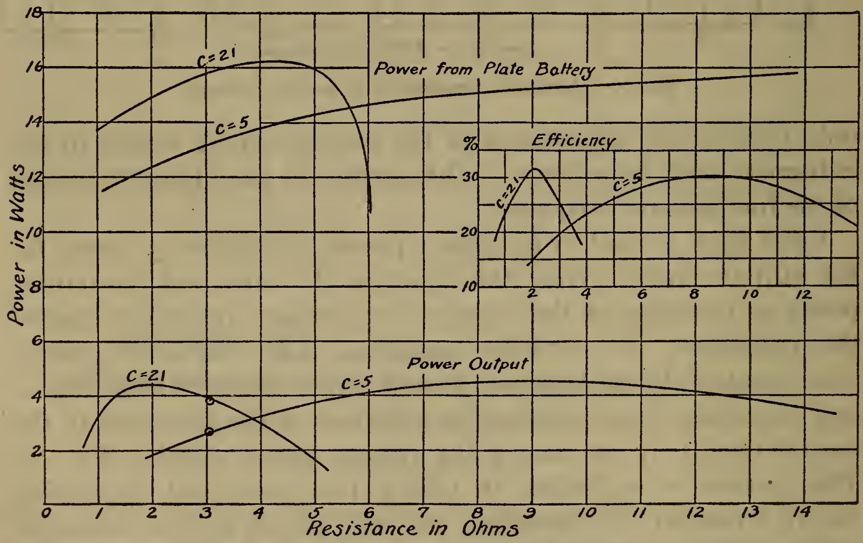

FIG. 11.—*Power output—Constant capacity*

Experimental work is now in progress upon a method for obtaining by direct measurement the current characteristics of power tubes, as shown on Fig. 4. This will make unnecessary the use of harmonic analysis of the derived characteristics. The tube under test is excited by 60-cycle sinusoidal grid and plate voltages and mechanical resonance with this frequency is attained with a vibrating system which gives direct readings of the amplitudes of the fundamental constituents of the pulsating plate and grid currents. The power output from a tube can thus be predetermined as a function of n and of ${}_1E_g$ for different operating conditions of filament emission and plate voltage. The results of such measurements for a variety of tubes will be given in a later paper.

The author desires to express appreciation of the assistance of H. A. Snow of this Bureau who made most of the measurements described in this paper.

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